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Reg. No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

VI Semester B.Sc. Degree Examination, August/September - 2023
MATHEMATICS
(CBCS Scheme)
Paper : VIII

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates :

Answer **ALL** questions.

I. Answer any **FIVE** questions. **(5×2=10)**

1. Show that $\arg\left(\frac{\bar{z}}{z}\right) = \frac{\pi}{2}$ represents a line through the origin.

2. Define continuity of $f(z)$ at the point $z = z_0$.

3. Define harmonic function, give an example.

4. Verify that $f(z) = u + iv$ where $u = x^2 - y^2$ and $v = 2xy$ are the real and imaginary parts of $f(z)$ is an analytic function.

5. Evaluate $\int_c (\bar{z})^2 dz$ around the circle $|z| = 1$.

6. Define cross ratio of four points Z_1, Z_2, Z_3, Z_4.

7. Find the real root of the equation $x^3 - x - 2 = 0$ in the interval $(1.5, 2)$ upto 2 approximations by bisection method.

8. State formula for Runge-Kutta method.

II. Answer any **Three** questions. **(3×5=15)**

9. Find the locus of the point z satisfying $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$

10. Evaluate $\lim_{z \to 2e^{i\pi/4}} \left(\frac{z^2 - 4}{z^2 + z + 5}\right)$

11. Find the orthogonal trajectories of the family of curves $x^3 y - xy^3 = 6$.

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12. State and Prove necessary condition for a function $f(z) = u(x, y) + iv(x, y)$ to be analytic.

13. Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.

III. Answer any **Three** questions. (3×5=15)

14. Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along $y = x^2 + 1$.

15. State and Prove Cauchy's integral theorem.

16. Evaluate $\int_c \frac{e^z}{z(z-2)} dz$ where c is $|z| = 3$.

17. Discuss the transformation $w = \sinh z$.

18. Prove that the bilinear transformation preserves the cross ratio of four points.

IV. Answer any **Three** questions. (3×5=15)

19. Using Newton Raphson method, find the real root of the equation $x^3 + 5x - 11 = 0$ by performing 3 iterations only.

20. Solve the equations by Gauss Seidel method.

$$10x + 2y + z = 9,\ x + 10y - z = -22,\ -2x + 3y + 10z = 22$$

21. Find the largest eigen value of the matrix $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by power method.

22. Using Taylor series method, solve $\frac{dy}{dx} = x^2 y - 1$, $y(0) = 1$, find $y(0.1)$ correct to 3 decimals taking upto 3rd degree term.

23. Using Runge-Kutta method, solve $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0.4) = 1$ at $x = 0.5$ correct to 3 decimal places.

V. Answer any **Three** questions. (3×5=15)

24. A two dimensional flow field is given by $\psi = xy$. Show that flow is irrotational. Find the stream lines and potential lines.

25. Expand $\frac{1}{z+1}$ about $z = 1$, in Taylor's series.

26. Show that $u = -wy,\ v = wx,\ w = 0$ represents a possible motion of inviscid fluid. Find the stream function and sketch stream lines.

27. The concentration of salt x in a home made soap maker is given as a function of time by $\frac{dx}{dt} = 37.5 - 3.5x$. At the initial time $t = 0$, the salt concentration in the tank is $50\ g/L$. Using Runge Kutta method with a step size of $h = 1.5$ min., what is the salt concentration after 1.5 minutes.

28. A polluted lake has an initial concentration of bacteria of $10^7\ parts/m^3$ while the acceptable level $5\times10^6\ parts/m^3$. The concentration of the bacteria will reduce as fresh waters enter the lake. The differential equation that governs the concentration C of the pollutant as a function of time (in weeks) is given by $\frac{dc}{dt} + 0.06\,C = 0 \quad C(0) = 10$. Using Euler's method and a step size of 3.5 weeks, find the concentration of the pollutant after 7 weeks.